GCSE Maths – Number

Powers, Roots and Fractional Indices

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of questions on powers, roots and fractional indices. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example 1

Find 13^3

Step 1: Identify the power and use the number to indicate how many multiples there are.

$$13^3 = 13 \times 13 \times 13$$

Step 2: Calculate the product.

$$13^3 = 13 \times 13 \times 13 = 2197$$

Worked Example 2

Find $\left(\frac{5}{4}\right)^{-4}$

Step 1: Due to the negative sign, flip the base.

$$\left(\frac{5}{4}\right)^{-4} = \left(\frac{4}{5}\right)^{4}$$

Step 2: Apply the remaining power to the numerator and denominator.

$$\left(\frac{5}{4}\right)^{-4} = \left(\frac{4}{5}\right)^{4} = \frac{4^4}{5^4} = \frac{4 \times 4 \times 4 \times 4}{5 \times 5 \times 5 \times 5} = \frac{256}{625}$$

Guided Example 1

Find 21^2

Step 1: Identify the power and use the number to indicate how many multiples there are.

$$21^2 = 21 \times 21$$

Step 2: Calculate the product.

$$21^2 = 21 \times 21 = 441$$

$$\begin{array}{r} 21 \\ \times\ 21 \\ \hline 21 \\ +\ 420 \\ \hline 441 \end{array}$$

Guided Example 2

Find $\left(\frac{13}{3}\right)^{-2}$

Step 1: Due to the negative sign, flip the base.

$$\left(\frac{13}{3}\right)^{-2} = \left(\frac{3}{13}\right)^{2}$$

Step 2: Apply the remaining power to the numerator and denominator.

$$\left(\frac{3}{13}\right)^{2} = \frac{3^2}{13^2} = \frac{3 \times 3}{13 \times 13} = \frac{9}{169}$$

$$\begin{array}{r} 13 \\ \times\ 13 \\ \hline 39 \\ +\ 130 \\ \hline 169 \end{array}$$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

1. Find 7^3

$7^3 = 7 \times 7 \times 7 = 49 \times 7 = 343$

$49 \times 7 = 343$

2. Find 4^4

$4^4 = 4 \times 4 \times 4 \times 4 = 16 \times 4 \times 4 = 64 \times 4 = 256$

$64 \times 4 = 256$

3. Find 5^6

$5^6 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 25 \times 5 \times 5 \times 5 \times 5 = 125 \times 5 \times 5 \times 5 = 625 \times 5 \times 5$
$= 3125 \times 5$
$= 15625$

$3125 \times 5 = 15625$

4. Find $\left(\frac{3}{5}\right)^2$

$\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2} = \frac{3 \times 3}{5 \times 5} = \frac{9}{25}$

5. Find $\left(\frac{-9}{4}\right)^3$

$\left(\frac{-9}{4}\right)^3 = \frac{(-9)^3}{4^3} = \frac{-9 \times -9 \times -9}{4 \times 4 \times 4} = \frac{-729}{64}$

6. Find 2^{-3}

$2^{-3} = \left(\frac{2}{1}\right)^{-3} = \left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3} = \frac{1}{8}$

flip the base to remove negative index

7. Find 0.5^5

$0.5^5 = \left(\frac{1}{2}\right)^5 = \frac{1^5}{2^5} = \frac{1}{32}$

convert decimal to fraction

8. Find $\left(\frac{7}{11}\right)^{-4}$

$\left(\frac{7}{11}\right)^{-4} = \left(\frac{11}{7}\right)^4 = \frac{11^4}{7^4} = \frac{11 \times 11 \times 11 \times 11}{7 \times 7 \times 7 \times 7} = \frac{14641}{2401}$

flip the base to remove negative index

9. Find $\left(\frac{2}{3}\right)^7$

$\left(\frac{2}{3}\right)^7 = \frac{2^7}{3^7} = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3} = \frac{128}{2187}$

10. Find $(-11)^{-4}$

flip the base to remove negative index

$(-11)^{-4} = \left(\frac{-11}{1}\right)^{-4} = \left(\frac{1}{-11}\right)^4 = \frac{1^4}{(-11)^4} = \frac{1}{11^4} = \frac{1}{11 \times 11 \times 11 \times 11} = \frac{1}{14641}$

$(-11)^4 = (-1)^4(11)^4 = 11^4$

Section B

Worked Example 1

Simplify $g^5 \times g^3$

Step 1: As we are multiplying, we must add the two powers together.

$$g^5 \times g^3 = g^{5+3}$$

Step 2: Simplify the addition of the two powers.

$$g^{5+3} = g^8$$

Worked Example 2

Simplify $(q^6)^{11}$

Step 1: As we are raising a power to another power, we must multiply the two powers together.

$$(q^6)^{11} = q^{6\times 11}$$

Step 2: Simplify the multiplication of the two powers.

$$q^{6\times 11} = q^{66}$$

Guided Example

Simplify $y^9 \div y^{\frac{1}{2}}$

Step 1: As we are dividing, we must subtract the second powers from the first power.

$$y^9 \div y^{\frac{1}{2}} = y^{9-\frac{1}{2}}$$

RULES OF INDICES:

$x^m \div x^n = x^{m-n}$

Step 2: Simplify the subtraction of the two powers.

$$y^{9-\frac{1}{2}} = y^{\frac{18}{2}-\frac{1}{2}} = y^{\frac{17}{2}}$$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

11. Simplify $x \times x \times x$

$x \times x \times x = x^{1+1+1} = x^3$

12. Simplify $a^3 \times a^4$

$a^3 \times a^4 = a^{3+4} = a^7$

13. Simplify $r^{40} \div r^{21}$

$r^{40} \div r^{21} = r^{40-21} = r^{19}$

14. Simplify $e^{\frac{3}{4}} \times e^{\frac{1}{2}}$

$e^{\frac{3}{4}} \times e^{\frac{1}{2}} = e^{\frac{3}{4}+\frac{1}{2}} = e^{\frac{3}{4}+\frac{2}{4}} = e^{\frac{5}{4}}$

15. Simplify $t^{\frac{7}{3}} \div t^2$

$t^{\frac{7}{3}} \div t^2 = t^{\frac{7}{3}-2} = t^{\frac{7}{3}-\frac{6}{3}} = t^{\frac{1}{3}}$

16. Simplify $(a^2)^3$

$(a^2)^3 = a^{2\times3} = a^6$

17. Simplify $(9b^4)^7$

$(9b^4)^7 = 9^7 b^{4\times7} = 4782969\, b^{28}$

18. Simplify $(3f^5)^{\frac{9}{10}}$

$(3f^5)^{\frac{9}{10}} = 3^{\frac{9}{10}} (f^5)^{\frac{9}{10}} = 3^{\frac{9}{10}} f^{5\times\frac{9}{10}} = \sqrt[10]{3^9}\, f^{\frac{45}{10}} = \sqrt[10]{19683}\, f^{\frac{9}{2}}$

19. Simplify $(p^{-q})^{-r}$

$(p^{-q})^{-r} = p^{-q\times-r} = p^{qr}$

20. Simplify $\left(\frac{x^{2y}}{x^y}\right)^3$

$\left(\frac{x^{2y}}{x^y}\right)^3 = \frac{(x^{2y})^3}{(x^y)^3} = \frac{x^{2y\times3}}{x^{y\times3}} = \frac{x^{6y}}{x^{3y}} = x^{6y} \div x^{3y} = x^{6y-3y} = x^{3y}$

RULES OF INDICES

1. $x^m \times x^n = x^{m+n}$
2. $x^m \div x^n = x^{m-n}$
3. $(x^m)^n = x^{m\times n}$
4. $x^{-n} = \frac{1}{x^n}$
5. $x^{\frac{n}{m}} = \sqrt[m]{x^n}$

Section C – Higher Only

Worked Example 1

Find and simplify $\sqrt{68}$

Step 1: Identify if the number in the root has any square number factors.

$68 = 4 \times 17$ *so* 4 *is a square number factor.*

Step 2: Simplify the square root using rules of surds.

$$\sqrt{68} = \sqrt{4 \times 17} = \sqrt{4} \times \sqrt{17} = 2 \times \sqrt{17} = \mathbf{2\sqrt{17}}$$

Worked Example 2

Find and simplify $\sqrt[4]{625}$

Step 1: Without using a calculator, find an integer which factors into 625 exactly 4 times (the same number of times as the root).

$$625 = 5 \times 5 \times 5 \times 5 = 5^4$$

Step 2: Deduce the solution to the root expression.

$$\sqrt[4]{625} = \sqrt[4]{5^4} = \mathbf{5}$$

Guided Example 1

Find and simplify $\sqrt{126}$

Step 1: Identify if the number in the root has any square number factors.

$126 = 9 \times 14$ so 9 is a square number factor.

Step 2: Simplify the square root using rules of surds.

$$\sqrt{126} = \sqrt{9 \times 14} = \sqrt{9} \times \sqrt{14} = 3\sqrt{14}$$

rules of surds

Guided Example 2

Find and simplify $\sqrt[5]{32}$

Step 1: Without using a calculator, find an integer which factors into 32 exactly 5 times (the same number of times as the root).

$$32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$$

Step 2: Deduce the solution to the root expression.

$$\sqrt[5]{32} = \sqrt[5]{2^5} = 2$$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

21. Find $\sqrt{81}$

$81 = 9 \times 9$ so 81 has two square number factors of 9.

$\sqrt{81} = \sqrt{9 \times 9} = \sqrt{9} \times \sqrt{9} = 3 \times 3 = 9$

Alternative method: $81 = 9 \times 9 = 9^2$

$\sqrt{81} = \sqrt{9^2} = 9$

22. Find $\sqrt{24}$

$2^2 = 4$

$24 = 6 \times 4$ so 4 is a square number factor.

$\sqrt{24} = \sqrt{6 \times 4} = \sqrt{6} \times \sqrt{4} = \sqrt{6} \times 2 = 2\sqrt{6}$

23. Find $\sqrt{900}$

$10^2 = 100$

$900 = 100 \times 9$ so 100 is a square number factor.

$\sqrt{900} = \sqrt{100 \times 9} = \sqrt{100} \times \sqrt{9} = 10 \times 3 = 30$

Alternative method:

$900 = 30 \times 30 = 30^2$

$\sqrt{900} = \sqrt{30^2} = 30$

24. Find $\sqrt{612}$

$612 = 36 \times 17$ so 36 is a square number factor.

$\sqrt{612} = \sqrt{36 \times 17} = \sqrt{36} \times \sqrt{17} = 6 \times \sqrt{17} = 6\sqrt{17}$

25. Find $2\sqrt{128}$

$128 = 64 \times 2$ so 64 is a square number factor.

$\sqrt{128} = \sqrt{64 \times 2} = \sqrt{64} \times \sqrt{2} = 8 \times \sqrt{2} = 8\sqrt{2}$

$2\sqrt{128} = 2(8\sqrt{2}) = 16\sqrt{2}$

26. Find $13\sqrt{338}$

$169 = 13^2$

$338 = 169 \times 2$ so 169 is a square number factor.

$\sqrt{338} = \sqrt{169 \times 2} = \sqrt{169} \times \sqrt{2} = 13 \times \sqrt{2} = 13\sqrt{2}$

$13\sqrt{338} = 13(13\sqrt{2}) = 169\sqrt{2}$

27. Find $\sqrt[3]{64}$

$64 = 16 \times 4 = 4 \times 4 \times 4 = 4^3$

$\sqrt[3]{64} = \sqrt[3]{4^3} = 4$

28. Find $\sqrt[4]{16}$

$16 = 4 \times 4 = 2 \times 2 \times 2 \times 2 = 2^4$

$\sqrt[4]{16} = \sqrt[4]{2^4} = 2$

29. Find $\sqrt[3]{125}$

$125 = 25 \times 5 = 5 \times 5 \times 5 = 5^3$

$\sqrt[3]{125} = \sqrt[3]{5^3} = 5$

30. Find $\sqrt[5]{243}$

$243 = 81 \times 3 = 9 \times 9 \times 3 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$

$\sqrt[5]{243} = \sqrt[5]{3^5} = 3$

Worked Example

Find and simplify $2^{-\frac{3}{2}}$

Step 1: Due to the negative sign, flip the base.

$$2^{-\frac{3}{2}} = \left(\frac{1}{2}\right)^{\frac{3}{2}}$$

Step 2: Apply the remaining index to the numerator and denominator.

$$\left(\frac{1}{2}\right)^{\frac{3}{2}} = \frac{1^{\frac{3}{2}}}{2^{\frac{3}{2}}}$$

Step 3: Simplify the remaining powers and roots. Rationalise the denominator if necessary.

$$\frac{1^{\frac{3}{2}}}{2^{\frac{3}{2}}} = \frac{1}{\sqrt{2^3}} = \frac{1}{\sqrt{8}} = \frac{1}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}} = \frac{\sqrt{8}}{8} = \frac{\sqrt{4 \times 2}}{8} = \frac{\sqrt{4} \times \sqrt{2}}{8} = \frac{2\sqrt{2}}{8} = \frac{\sqrt{2}}{4}$$

Guided Example

Find and simplify $\left(\frac{4}{5}\right)^{\frac{1}{2}}$

Step 1: Apply the power to the numerator and denominator. As it is a fraction, we will get a root.

$$\left(\frac{4}{5}\right)^{\frac{1}{2}} = \frac{4^{\frac{1}{2}}}{5^{\frac{1}{2}}} = \frac{\sqrt{4}}{\sqrt{5}}$$

Step 2: Simplify the remaining powers and roots. Rationalise the denominator if necessary.

This is the same as multiplying by 1

$$\frac{\sqrt{4}}{\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

RATIONALISE THE DENOMINATOR

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

31. Find and simplify where possible $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9} = 3$

32. Find and simplify where possible $12^{-\frac{3}{2}}$

RATIONALISE THE DENOMINATOR

$12^{-\frac{3}{2}} = \left(\frac{12}{1}\right)^{-\frac{3}{2}} = \left(\frac{1}{12}\right)^{\frac{3}{2}} = \frac{1^{\frac{3}{2}}}{12^{\frac{3}{2}}} = \frac{\sqrt{1^3}}{\sqrt{12^3}} = \frac{1}{\sqrt{1728}} = \frac{1}{\sqrt{1728}} \times \frac{\sqrt{1728}}{\sqrt{1728}} = \frac{\sqrt{1728}}{1728} =$

$= \frac{\sqrt{576 \times 3}}{1728} = \frac{\sqrt{576} \times \sqrt{3}}{1728} = \frac{\sqrt{24^2} \times \sqrt{3}}{1728} = \frac{24\sqrt{3}}{1728} = \frac{\sqrt{3}}{72}$

divide numerator and denominator by 24

33. Find and simplify where possible $\left(\frac{4}{3}\right)^{\frac{5}{2}}$

RATIONALISE THE DENOMINATOR

$\left(\frac{4}{3}\right)^{\frac{5}{2}} = \frac{4^{\frac{5}{2}}}{3^{\frac{5}{2}}} = \frac{\sqrt{4^5}}{\sqrt{3^5}} = \frac{(\sqrt{4})^5}{\sqrt{3^5}} = \frac{2^5}{\sqrt{3^5}} = \frac{2 \times 2 \times 2 \times 2 \times 2}{\sqrt{3 \times 3 \times 3 \times 3 \times 3}} = \frac{32}{\sqrt{243}} = \frac{32}{\sqrt{243}} \times \frac{\sqrt{243}}{\sqrt{243}} =$

$= \frac{32\sqrt{243}}{243} = \frac{32\sqrt{81 \times 3}}{243} = \frac{32(\sqrt{81} \times \sqrt{3})}{243} = \frac{32(9 \times \sqrt{3})}{243} = \frac{32(9\sqrt{3})}{243} = \frac{288\sqrt{3}}{243} = \frac{32\sqrt{3}}{27}$

Simplify surd by finding square number factor of 243

divide numerator and denominator by 9

34. Find and simplify where possible $\left(\frac{27}{64}\right)^{-\frac{1}{3}}$

$\left(\frac{27}{64}\right)^{-\frac{1}{3}} = \left(\frac{64}{27}\right)^{\frac{1}{3}} = \frac{\sqrt[3]{64}}{\sqrt[3]{27}} = \frac{\sqrt[3]{4^3}}{\sqrt[3]{3^3}} = \frac{4}{3}$

Flip the base to remove negative index

35. Find and simplify where possible $9^{-\frac{1}{2}}$

$9^{-\frac{1}{2}} = \left(\frac{9}{1}\right)^{-\frac{1}{2}} = \left(\frac{1}{9}\right)^{\frac{1}{2}} = \frac{1^{\frac{1}{2}}}{9^{\frac{1}{2}}} = \frac{\sqrt{1}}{\sqrt{9}} = \frac{1}{3}$

Flip the base to remove negative index

36. Find and simplify where possible $\left(\frac{7}{8}\right)^{-\frac{2}{3}}$

$$\left(\frac{7}{8}\right)^{-\frac{2}{3}} = \left(\frac{8}{7}\right)^{\frac{2}{3}} = \frac{8^{\frac{2}{3}}}{7^{\frac{2}{3}}} = \frac{\sqrt[3]{8^2}}{\sqrt[3]{7^2}} = \frac{\sqrt[3]{64}}{\sqrt[3]{49}} = \frac{\sqrt[3]{4^3}}{\sqrt[3]{49}} = \frac{4}{\sqrt[3]{49}}$$

Flip the base to remove negative index

This solution cannot be further simplified since denominators with cube roots can not be rationalised in the same way as denominators with square roots.

CHALLENGE: How could you adapt the square root rationalisation method for cube roots?

37. Find and simplify where possible $\left(\frac{16}{81}\right)^{-\frac{5}{4}}$

$$\left(\frac{16}{81}\right)^{-\frac{5}{4}} = \left(\frac{81}{16}\right)^{\frac{5}{4}} = \frac{81^{\frac{5}{4}}}{16^{\frac{5}{4}}} = \frac{\sqrt[4]{81^5}}{\sqrt[4]{16^5}} = \frac{(\sqrt[4]{81})^5}{(\sqrt[4]{16})^5} = \frac{(\sqrt[4]{3^4})^5}{(\sqrt[4]{2^4})^5} = \frac{3^5}{2^5} = \frac{3\times3\times3\times3\times3}{2\times2\times2\times2\times2} = \frac{243}{32}$$

Flip the base to remove negative index

You can apply the power and root in whichever order is simplest

38. Find and simplify where possible $\left(\frac{8}{27}\right)^{-\frac{4}{3}}$

$$\left(\frac{8}{27}\right)^{-\frac{4}{3}} = \left(\frac{27}{8}\right)^{\frac{4}{3}} = \frac{27^{\frac{4}{3}}}{8^{\frac{4}{3}}} = \frac{\sqrt[3]{27^4}}{\sqrt[3]{8^4}} = \frac{(\sqrt[3]{27})^4}{(\sqrt[3]{8})^4} = \frac{(\sqrt[3]{3^3})^4}{(\sqrt[3]{2^3})^4} = \frac{3^4}{2^4} = \frac{3\times3\times3\times3}{2\times2\times2\times2} = \frac{81}{16}$$

Flip the base to remove negative index

You can apply the power and root in whichever order is simplest → Here, it makes sense to apply the root first so we are handling smaller numbers

39. Find and simplify where possible $\left(\frac{9}{16}\right)^{-\frac{3}{2}}$

$$\left(\frac{9}{16}\right)^{-\frac{3}{2}} = \left(\frac{16}{9}\right)^{\frac{2}{3}} = \frac{16^{\frac{2}{3}}}{9^{\frac{2}{3}}} = \frac{\sqrt[3]{16^2}}{\sqrt[3]{9^2}} = \frac{\sqrt[3]{256}}{\sqrt[3]{81}}$$

This solution cannot be further simplified

Flip the base to remove negative index

40. Find and simplify where possible $(32)^{-\frac{2}{5}}$

$$(32)^{-\frac{2}{5}} = \left(\frac{32}{1}\right)^{-\frac{2}{5}} = \left(\frac{1}{32}\right)^{\frac{2}{5}} = \frac{1^{\frac{2}{5}}}{32^{\frac{2}{5}}} = \frac{\sqrt[5]{1^2}}{\sqrt[5]{32^2}} = \frac{1}{(\sqrt[5]{32})^2} = \frac{1}{(\sqrt[5]{2^5})^2} = \frac{1}{2^2} = \frac{1}{4}$$

Flip the base to remove negative index

You can apply the power and root in whichever order is simplest → Here, it makes sense to apply the root first so we are handling smaller numbers (and we know the 5th root of 32)

Section E – Higher Only

Worked Example

Estimate 6.5^2

Step 1: Recognise that 6.5 is between two integers, 6 and 7.

$$6 < 6.5 < 7$$

Step 2: Due to this, 6.5^2 is between 6^2 and 7^2.

$$6^2 < 6.5^2 < 7^2$$

Step 3: Simplify this inequality.

$$36 < 6.5^2 < 49$$

Step 3: Using this we can estimate 6.5^2.

$$6.5^2 \simeq 40$$

Guided Example

Estimate $\sqrt{14}$

Step 1: Recognise that 14 is between two square numbers, 9 and 16.

$$9 < 14 < 16$$

Step 2: Due to this the square root of 14 lies between the square root of 9 and the square root of 16.

$$\sqrt{9} < \sqrt{14} < \sqrt{16}$$

Step 3: As 14 is closer to 16 than to 9, square root of 14 is close to the square root of 16.

$$\sqrt{9} < \sqrt{14} < \sqrt{16}$$

$$3 < \sqrt{14} < 4$$

since $\sqrt{14}$ closer to $\sqrt{16}$ than $\sqrt{9}$, $\sqrt{14}$ is closer to 4 than 3. So we can estimate

$$\sqrt{14} \approx 3.7$$

For questions like this a range of answers will be allowed - as long as you select a value in the correct region!

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

For questions like the following, a range of answers would be allowed. You just need to make sure you're choosing a value in the correct region.

41. Estimate 4.3^2

$4 < 4.3 < 5$
$4^2 < 4.3^2 < 5^2$
$16 < 4.3^2 < 25$
4.3 is closer to 4 than 5 so 4.3^2 is closer to 16 than 25.
So, we estimate $4.3^2 \approx 19$

42. Estimate 1.4^3

$1 < 1.4 < 2$
$1^3 < 1.4^3 < 2^3$
$1 < 1.4^3 < 8$
1.4 is closer to 1 than 2 so 1.4^3 is closer to 1 than to 8.
So, we estimate $1.4^3 \approx 3$

43. Estimate 2.1^5

$2 < 2.1 < 3$
$2^5 < 2.1^5 < 3^5$
$32 < 2.1^5 < 243$
2.1 is closer to 2 than 3 so 2.1^2 will be closer to 32 than 243.
So, we estimate $2.1^5 \approx 40$

44. Estimate 0.823^2

$0 < 0.823 < 1$
$0^2 < 0.823^2 < 1^2$
$0 < 0.823^2 < 1$
0.823 is closer to 1 than 0 so 0.823^2 is closer to 1 than 0.
So, we estimate $0.823^2 \approx 0.7$

45. Estimate $\sqrt{39}$

$36 < 39 < 49$ ← Sandwich the required value between known square numbers.
$\sqrt{36} < \sqrt{39} < \sqrt{49}$
$6 < \sqrt{39} < 7$
39 is closer to 36 so $\sqrt{39}$ is closer to 6 than 7.
So, we estimate $\sqrt{39} \approx 6.2$

46. Estimate $\sqrt{35}$

$25 < 35 < 36$
$\sqrt{25} < \sqrt{35} < \sqrt{36}$
$5 < \sqrt{35} < 6$
35 is closer to 36 than 25 so $\sqrt{35}$ is closer to 6 than 5.
So, we estimate $\sqrt{35} \approx 5.9$

47. Estimate $\sqrt{140}$

$121 < 140 < 144$
$\sqrt{121} < \sqrt{140} < \sqrt{144}$
$11 < \sqrt{140} < 12$
140 is closer to 144 than 121 so $\sqrt{144}$ is closer to 12 than 11.
So, we estimate $\sqrt{140} \approx 11.8$

Alternative method: $\sqrt{140} = 2\sqrt{35}$
From Q46, $\sqrt{35} \approx 5.9$.
So $\sqrt{140} = 2\sqrt{35} \approx 2 \times 5.9 = 11.8$

48. Estimate $\sqrt{18.2}$

$16 < 18.2 < 25$
$\sqrt{16} < \sqrt{18.2} < \sqrt{25}$
$4 < \sqrt{18.2} < 5$
18.2 is closer to 16 than 25 so $\sqrt{18.2}$ is closer to 4 than 5.
So, we estimate $\sqrt{18.2} \approx 4.3$

49. Estimate $\sqrt[3]{61}$

$27 < 61 < 64$ ← Here we sandwich the required value between cube numbers due to the cube root.
$\sqrt[3]{27} < \sqrt[3]{61} < \sqrt[3]{64}$
$3 < \sqrt[3]{61} < 4$
61 is closer to 64 than 27 so $\sqrt[3]{61}$ is closer to 4 than 3.
So, we estimate $\sqrt[3]{61} \approx 3.9$